Event calculus axioms used in the book *Commonsense Reasoning* (Mueller, 2006)

Continuous Event Calculus (EC) (Miller & Shanahan, 2002)

EC1. $\mathit{Clipped}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 \leq t < t_2 \wedge \mathit{Terminates}(e, f, t))$

EC2. $\mathit{Declipped}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 \leq t < t_2 \wedge \mathit{Initiates}(e, f, t))$

EC3. $\mathit{StoppedIn}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 < t < t_2 \wedge \mathit{Terminates}(e, f, t))$

EC4. $\mathit{StartedIn}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 < t < t_2 \wedge \mathit{Initiates}(e, f, t))$

EC5. $(\mathit{Happens}(e, t_1) \wedge \mathit{Initiates}(e, f_1, t_1) \wedge 0 < t_2 \wedge \mathit{Trajectory}(f_1, t_1, f_2, t_2) \wedge \neg \mathit{StoppedIn}(t_1, f_1, t_1 + t_2)) \Rightarrow \mathit{HoldsAt}(f_2, t_1 + t_2)$

EC6. $(\mathit{Happens}(e, t_1) \wedge \mathit{Terminates}(e, f_1, t_1) \wedge 0 < t_2 \wedge \mathit{AntiTrajectory}(f_1, t_1, f_2, t_2) \wedge \neg \mathit{StartedIn}(t_1, f_1, t_1 + t_2)) \Rightarrow \mathit{HoldsAt}(f_2, t_1 + t_2)$

EC7. $\mathit{PersistsBetween}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \neg \exists t\, (\mathit{ReleasedAt}(f, t) \wedge t_1 < t \leq t_2)$

EC8. $\mathit{ReleasedBetween}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 \leq t < t_2 \wedge \mathit{Releases}(e, f, t))$

EC9. $(\mathit{HoldsAt}(f, t_1) \wedge t_1 < t_2 \wedge \mathit{PersistsBetween}(t_1, f, t_2) \wedge \neg \mathit{Clipped}(t_1, f, t_2)) \Rightarrow \mathit{HoldsAt}(f, t_2)$

EC10. $(\neg \mathit{HoldsAt}(f, t_1) \wedge t_1 < t_2 \wedge \mathit{PersistsBetween}(t_1, f, t_2) \wedge \neg \mathit{Declipped}(t_1, f, t_2)) \Rightarrow \neg \mathit{HoldsAt}(f, t_2)$

EC11. $(\mathit{ReleasedAt}(f, t_1) \wedge t_1 < t_2 \wedge \neg \mathit{Clipped}(t_1, f, t_2) \wedge \neg \mathit{Declipped}(t_1, f, t_2)) \Rightarrow \mathit{ReleasedAt}(f, t_2)$

EC12. $(\neg \mathit{ReleasedAt}(f, t_1) \wedge t_1 < t_2 \wedge \neg \mathit{ReleasedBetween}(t_1, f, t_2)) \Rightarrow \neg \mathit{ReleasedAt}(f, t_2)$

EC13. $\mathit{ReleasedIn}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 < t < t_2 \wedge \mathit{Releases}(e, f, t))$

EC14. $(\mathit{Happens}(e, t_1) \wedge \mathit{Initiates}(e, f, t_1) \wedge t_1 < t_2 \wedge \neg \mathit{StoppedIn}(t_1, f, t_2) \wedge \neg \mathit{ReleasedIn}(t_1, f, t_2)) \Rightarrow \mathit{HoldsAt}(f, t_2)$

EC15. $(\mathit{Happens}(e, t_1) \wedge \mathit{Terminates}(e, f, t_1) \wedge t_1 < t_2 \wedge \neg \mathit{StartedIn}(t_1, f, t_2) \wedge \neg \mathit{ReleasedIn}(t_1, f, t_2)) \Rightarrow \neg \mathit{HoldsAt}(f, t_2)$

EC16. $(\mathit{Happens}(e, t_1) \wedge \mathit{Releases}(e, f, t_1) \wedge t_1 < t_2 \wedge \neg \mathit{StoppedIn}(t_1, f, t_2) \wedge \neg \mathit{StartedIn}(t_1, f, t_2)) \Rightarrow \mathit{ReleasedAt}(f, t_2)$

EC17. $(\mathit{Happens}(e, t_1) \wedge (\mathit{Initiates}(e, f, t_1) \vee \mathit{Terminates}(e, f, t_1)) \wedge t_1 < t_2 \wedge \neg \mathit{ReleasedIn}(t_1, f, t_2)) \Rightarrow \neg \mathit{ReleasedAt}(f, t_2)$

Discrete Event Calculus (DEC) (Mueller, 2004)

DEC1. $\mathit{StoppedIn}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 < t < t_2 \wedge \mathit{Terminates}(e, f, t))$

DEC2. $\mathit{StartedIn}(t_1, f, t_2) \stackrel{\text{def}}{\equiv} \exists e, t\, (\mathit{Happens}(e, t) \wedge t_1 < t < t_2 \wedge \mathit{Initiates}(e, f, t))$

DEC3. $(\mathit{Happens}(e, t_1) \wedge \mathit{Initiates}(e, f_1, t_1) \wedge 0 < t_2 \wedge \mathit{Trajectory}(f_1, t_1, f_2, t_2) \wedge \neg \mathit{StoppedIn}(t_1, f_1, t_1 + t_2)) \Rightarrow \mathit{HoldsAt}(f_2, t_1 + t_2)$

DEC4. $(\mathit{Happens}(e, t_1) \wedge \mathit{Terminates}(e, f_1, t_1) \wedge 0 < t_2 \wedge \mathit{AntiTrajectory}(f_1, t_1, f_2, t_2) \wedge \neg \mathit{StartedIn}(t_1, f_1, t_1 + t_2)) \Rightarrow \mathit{HoldsAt}(f_2, t_1 + t_2)$

DEC5. $(\mathit{HoldsAt}(f, t) \wedge \neg \mathit{ReleasedAt}(f, t+1) \wedge \neg \exists e\, (\mathit{Happens}(e, t) \wedge \mathit{Terminates}(e, f, t))) \Rightarrow \mathit{HoldsAt}(f, t+1)$

DEC6. $(\neg \mathit{HoldsAt}(f, t) \wedge \neg \mathit{ReleasedAt}(f, t+1) \wedge \neg \exists e\, (\mathit{Happens}(e, t) \wedge \mathit{Initiates}(e, f, t))) \Rightarrow \neg \mathit{HoldsAt}(f, t+1)$

DEC7. $(\mathit{ReleasedAt}(f, t) \wedge \neg \exists e\, (\mathit{Happens}(e, t) \wedge (\mathit{Initiates}(e, f, t) \vee \mathit{Terminates}(e, f, t)))) \Rightarrow \mathit{ReleasedAt}(f, t+1)$

DEC8. $(\neg \mathit{ReleasedAt}(f, t) \wedge \neg \exists e\, (\mathit{Happens}(e, t) \wedge \mathit{Releases}(e, f, t))) \Rightarrow \neg \mathit{ReleasedAt}(f, t+1)$

DEC9. $(\mathit{Happens}(e, t) \wedge \mathit{Initiates}(e, f, t)) \Rightarrow \mathit{HoldsAt}(f, t+1)$

DEC10. $(\mathit{Happens}(e, t) \wedge \mathit{Terminates}(e, f, t)) \Rightarrow \neg \mathit{HoldsAt}(f, t+1)$

DEC11. $(\mathit{Happens}(e, t) \wedge \mathit{Releases}(e, f, t)) \Rightarrow \mathit{ReleasedAt}(f, t+1)$

DEC12. $(\mathit{Happens}(e, t) \wedge (\mathit{Initiates}(e, f, t) \vee \mathit{Terminates}(e, f, t))) \Rightarrow \neg \mathit{ReleasedAt}(f, t+1)$

Causal Constraints (CC) (Shanahan, 1999)

CC1. $\mathit{Started}(f, t) \Leftrightarrow (\mathit{HoldsAt}(f, t) \vee \exists e\, (\mathit{Happens}(e, t) \wedge \mathit{Initiates}(e, f, t)))$

CC2. $\mathit{Stopped}(f, t) \Leftrightarrow (\neg \mathit{HoldsAt}(f, t) \vee \exists e\, (\mathit{Happens}(e, t) \wedge \mathit{Terminates}(e, f, t)))$

CC3. $\mathit{Initiated}(f, t) \Leftrightarrow (\mathit{Started}(f, t) \wedge \neg \exists e\, (\mathit{Happens}(e, t) \wedge \mathit{Terminates}(e, f, t)))$

CC4. $\mathit{Terminated}(f, t) \Leftrightarrow (\mathit{Stopped}(f, t) \wedge \neg \exists e\, (\mathit{Happens}(e, t) \wedge \mathit{Initiates}(e, f, t)))$

Events with Duration (Miller & Shanahan, 2002)

EC1′. $\mathit{Clipped}(t_1, f, t_4) \stackrel{\mathrm{def}}{\equiv} \exists e, t_2, t_3\, (\mathit{Happens3}(e, t_2, t_3) \wedge t_1 \leq t_3 \wedge t_2 < t_4 \wedge \mathit{Terminates}(e, f, t_2))$

EC2′. $\mathit{Declipped}(t_1, f, t_4) \stackrel{\mathrm{def}}{\equiv} \exists e, t_2, t_3\, (\mathit{Happens3}(e, t_2, t_3) \wedge t_1 \leq t_3 \wedge t_2 < t_4 \wedge \mathit{Initiates}(e, f, t_2))$

EC3′. $\mathit{StoppedIn}(t_1, f, t_4) \stackrel{\mathrm{def}}{\equiv} \exists e, t_2, t_3\, (\mathit{Happens3}(e, t_2, t_3) \wedge t_1 < t_3 \wedge t_2 < t_4 \wedge \mathit{Terminates}(e, f, t_2))$

EC4′. $\mathit{StartedIn}(t_1, f, t_4) \stackrel{\mathrm{def}}{\equiv} \exists e, t_2, t_3\, (\mathit{Happens3}(e, t_2, t_3) \wedge t_1 < t_3 \wedge t_2 < t_4 \wedge \mathit{Initiates}(e, f, t_2))$

EC5′. $(\mathit{Happens3}(e, t_1, t_2) \wedge \mathit{Initiates}(e, f_1, t_1) \wedge 0 < t_3 \wedge \mathit{Trajectory}(f_1, t_1, f_2, t_3) \wedge \neg \mathit{StoppedIn}(t_1, f_1, t_2 + t_3)) \Rightarrow \mathit{HoldsAt}(f_2, t_2 + t_3)$

EC6′. $(\mathit{Happens3}(e, t_1, t_2) \wedge \mathit{Terminates}(e, f_1, t_1) \wedge 0 < t_3 \wedge \mathit{AntiTrajectory}(f_1, t_1, f_2, t_3) \wedge \neg \mathit{StartedIn}(t_1, f_1, t_2 + t_3)) \Rightarrow \mathit{HoldsAt}(f_2, t_2 + t_3)$

EC7. $\mathit{PersistsBetween}(t_1, f, t_2) \stackrel{\mathrm{def}}{\equiv} \neg \exists t\, (\mathit{ReleasedAt}(f, t) \wedge t_1 < t \leq t_2)$

EC8′. $\mathit{ReleasedBetween}(t_1, f, t_4) \stackrel{\mathrm{def}}{\equiv} \exists e, t_2, t_3\, (\mathit{Happens3}(e, t_2, t_3) \wedge t_1 \leq t_3 \wedge t_2 < t_4 \wedge \mathit{Releases}(e, f, t_2))$

EC9. $(\mathit{HoldsAt}(f, t_1) \wedge t_1 < t_2 \wedge \mathit{PersistsBetween}(t_1, f, t_2) \wedge \neg \mathit{Clipped}(t_1, f, t_2)) \Rightarrow \mathit{HoldsAt}(f, t_2)$

EC10. $(\neg \mathit{HoldsAt}(f, t_1) \wedge t_1 < t_2 \wedge \mathit{PersistsBetween}(t_1, f, t_2) \wedge \neg \mathit{Declipped}(t_1, f, t_2)) \Rightarrow \neg \mathit{HoldsAt}(f, t_2)$

EC11. $(\mathit{ReleasedAt}(f, t_1) \wedge t_1 < t_2 \wedge \neg \mathit{Clipped}(t_1, f, t_2) \wedge \neg \mathit{Declipped}(t_1, f, t_2)) \Rightarrow \mathit{ReleasedAt}(f, t_2)$

EC12. $(\neg \mathit{ReleasedAt}(f, t_1) \wedge t_1 < t_2 \wedge \neg \mathit{ReleasedBetween}(t_1, f, t_2)) \Rightarrow \neg \mathit{ReleasedAt}(f, t_2)$

EC13′. $\mathit{ReleasedIn}(t_1, f, t_4) \stackrel{\mathrm{def}}{\equiv} \exists e, t_2, t_3\, (\mathit{Happens3}(e, t_2, t_3) \wedge t_1 < t_3 \wedge t_2 < t_4 \wedge \mathit{Releases}(e, f, t_2))$

EC14′. $(\mathit{Happens3}(e, t_1, t_2) \wedge \mathit{Initiates}(e, f, t_1) \wedge t_2 < t_3 \wedge \neg \mathit{StoppedIn}(t_1, f, t_3) \wedge \neg \mathit{ReleasedIn}(t_1, f, t_3)) \Rightarrow \mathit{HoldsAt}(f, t_3)$

EC15′. $(\mathit{Happens3}(e, t_1, t_2) \wedge \mathit{Terminates}(e, f, t_1) \wedge t_2 < t_3 \wedge \neg \mathit{StartedIn}(t_1, f, t_3) \wedge \neg \mathit{ReleasedIn}(t_1, f, t_3)) \Rightarrow \neg \mathit{HoldsAt}(f, t_3)$

EC16′. $(\mathit{Happens3}(e, t_1, t_2) \wedge \mathit{Releases}(e, f, t_1) \wedge t_2 < t_3 \wedge \neg \mathit{StoppedIn}(t_1, f, t_3) \wedge \neg \mathit{StartedIn}(t_1, f, t_3)) \Rightarrow \mathit{ReleasedAt}(f, t_3)$

EC17′. $(\mathit{Happens3}(e, t_1, t_2) \wedge (\mathit{Initiates}(e, f, t_1) \vee \mathit{Terminates}(e, f, t_1)) \wedge t_2 < t_3 \wedge \neg \mathit{ReleasedIn}(t_1, f, t_3)) \Rightarrow \neg \mathit{ReleasedAt}(f, t_3)$

EC18. $\mathit{Happens3}(e, t_1, t_2) \Leftrightarrow t_1 \leq t_2$

EC19. $\mathit{Happens}(e, t) \Leftrightarrow \mathit{Happens3}(e, t, t)$

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